INSTITUTE OF MATHEMATICS & ITS APPLICATIONS

Trigonometry – Tutor Notes

This session builds on the concepts of trigonometry introduced in the prereading, providing additional examples and explanations to reinforce understanding. Begin by assessing students' current knowledge – ask what they recall about trigonometry from A-level or from the prereading. Use this to identify which sections may need clarification. If students express uncertainty about a particular topic, or indicate they found a section challenging, you can navigate directly to the relevant guidance below. The material is structured to follow the prereading sequence, allowing you to address specific gaps based on your student cohort's confidence levels.

Part I

Welcome Back – 15 Minutes

Degrees and radians: Begin by asking students to convert between degrees and radians. Work through examples: $90° = \pi/2$ rad, $225° = 5\pi/4$ rad, $540° = 3\pi$ rad, $30° = \pi/6$ rad. Then try the reverse: $\pi/4$ rad $= 45°$, 3π rad $= 540°$, $2\pi/6 = \pi/3$ rad $= 60°$. Emphasise that many engineering calculations require angles in radians – show students how to switch their calculator mode between degrees and radians, and how to check which mode is active.

Right-angled triangle trigonometry: Refer to the right-angled triangle in Section 2 of the prereading, and reintroduce SOH-CAH-TOA. Derive $\tan\theta = \sin\theta/\cos\theta$ by noting $\tan = \frac{O}{A} = \frac{O/H}{A/H} = \frac{\sin}{\cos}$. This is a good moment to define an *identity* – something true for all values the variable can take.

Pythagoras and distance formula: Remind students of the Pythagorean theorem with familiar triplets: $3, 4, 5$ and $5, 12, 13$ (and $6, 8, 10$ as a multiple). Derive the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ from $x^2 + y^2 = h^2$ – this is our second identity so far. As an extension, draw a line segment on a Cartesian plane and let students establish the horizontal and vertical differences, then apply Pythagoras to derive the distance formula.

The unit circle: Focus on the three key properties from the prereading:

- $\cos^2\theta + \sin^2\theta = 1$ – make the explicit link to the circle equation $x^2 + y^2 = r^2$ with $r = 1$.
- $\cos\theta$ and $\sin\theta$ range between -1 and 1. Label 1 and -1 on the axes, then pick points along the circle to show that coordinates never exceed 1. This visually reinforces the range.
- The signs of $\sin\theta$ and $\cos\theta$ depend on quadrant – this follows naturally from the previous point.

Return to the definition of radians: $\theta = \frac{\text{arc length}}{\text{radius}}$. On the unit circle ($r = 1$), the angle in radians equals the arc length. Use the three figures in the prereading to highlight the arcs for $90°$, $315°$, and $-45°$.

Exact values: Share any tips and tricks to aid memorising the exact values table. For $\cos\theta = 0$ at $\theta = 90°$ and $270°$, return to the circle definition – these are the points directly above and below the origin where the x-coordinate is zero.

Graphs of trig functions: Engage students with the properties of the graphs. Note that sine and cosine are simply a $90°$ shift apart – thus $\sin(\theta + 90°) = \cos\theta$. If helpful, express angles in both degrees and radians throughout the discussion.

Trigonometric identities: Start with the Pythagorean identity – we've already encountered it. Introduce compound angle formulas with a simple example: expand $\sin(30° + 45°)$ using $\sin(A + B) = \sin A \cos B + \cos A \sin B$. These are tools for simplifying expressions and solving equations. The sum-to-product formulas appear in problems like Q8 (gear tooth force) – reassure students they don't need to memorise everything immediately; these identities are provided in formula booklets across all major specification boards.

Small-angle approximations: Demonstrate with $\theta = 0.1$ rad ($\approx 5.7°$): $\sin 0.1 = 0.0998$, and $\tan 0.1 = 0.1003$. Explain that for small angles, these graphs can be approximated by straight lines of gradient 1 – so $\sin\theta \approx \theta$ and $\tan\theta \approx \theta$. This is why the approximations work.

Sine and cosine rules: Remind students that not all triangles are right-angled. Write both rules and clarify when each is used:

- Sine rule: two angles and a side, or two sides and a non-included angle.
- Cosine rule: two sides and the included angle, or three sides.

Note that when $A = 90°$, the cosine rule reduces to the familiar Pythagoras theorem.

Part II

Getting Started – 10 Minutes

Let students work on **Question 1**, then walk through the answers. Pay special attention to the equations formed – check that students correctly interpret "walks 95 m directly away from the river" as increasing the distance from the tower by 95 m, giving $d + 95$ in the second equation. Emphasise drawing a clear diagram showing the tower, the two observation points, and the two angles; a good diagram often clarifies the relationship between the variables. Watch for algebraic errors when solving for d – particularly when rearranging $d \tan 27° = (d+95) \tan 18°$. Remind students to keep more decimal places during intermediate calculations to avoid rounding errors, rounding only at the final step. The answer $h = 85$ m (to the nearest metre) is a good check – ask students whether this seems reasonable for the application.

Getting Stuck In – 30 Minutes

Ask students to focus on **Questions 2 to 4**. Pay special attention to:

- **Q2:** Elicit students to demonstrate that they understand the inherent symmetry of the function $y = 0.25x^2$ about the $x = 0$ line – this forces the strut PQ to be horizontal at $y = 4$. Students may need reminding that the gradient of a curve is given by the derivative $\frac{dy}{dx} = 0.5x$.
- **Q3:** This question links trigonometry to kinematics. Ensure students understand why the x-equation is linear while the y-equation is quadratic. A simple understanding check: "What is the velocity when the object hits the ground?" Any zero-answers tend to indicate a gap in understanding – take this opportunity to reiterate that only vertical motion has a force (weight); horizontal motion has no force, hence constant velocity. Vertically, the initial upward velocity decreases to zero at maximum height, then increases downward until impact. The horizontal motion throughout this journey is simply an application of speed = distance/time. Part (b) requires eliminating t to obtain the trajectory – a key algebraic skill. Part (c) involves exact values; students may need reminding how to rationalise denominators when simplifying expressions like $\frac{x^2}{2+\sqrt{3}}$.
- **Q4:** Appreciate that students may find this context more abstract and less tangible than previous ones – reassure them that the diagram in part (a) will guide the subsequent parts. Students often forget to convert units consistently (metres vs kilometres) – emphasise the importance of working in consistent units throughout. The approximation $\sqrt{2Rh}$ is worth highlighting; it's accurate enough for small h. Demonstrate to students that parts (c) and (d) are related – one requires finding d given h, the other requires finding h given d. Part (d) introduces the 8% refraction adjustment – a good example of how real-world factors modify theoretical models.

Break

Encourage students to step away from screens briefly.

Part III

More Problem Solving – 30 Minutes

Ask students to focus on **Questions 5 to 7** in that order.

- **Q5:** Students may need reminding that $\sec\theta = 1/\cos\theta$. When converting degrees to radians, point out that radian values will be much smaller than the corresponding degree measure – for example, $360° = 2\pi \approx 6.28$ rad, so an angle like 5° becomes approximately 0.0873 rad. The percentage error calculation in part (b) is good practice in comparing exact and approximate values. Part (e) requires numerical trial – encourage systematic testing (e.g., trying 2.5°, 3.0°, then refining) rather than guessing. Note that the coating thickness is given in microns (μm) – the prefix μ denotes 10^{-6}, so $d = 0.1\,\mu\text{m} = 0.1 \times 10^{-6}\,\text{m} = 10^{-7}\,\text{m}$.
- **Q6:** Ensure students understand that $AP^2 = 5 - 4\cos\theta$ comes from expanding $(2-\cos\theta)^2 + \sin^2\theta$ – a direct application of the distance formula. The condition $\cos\theta > 1/2$ leads to two intervals in $[0, 2\pi)$ – a good opportunity to discuss why cosine gives two solutions (one in the first quadrant, one in the fourth). It may also be worthwhile noting that the shaded region in the unit circle diagram makes complete sense when considering the receiver's position at $(2, 0)$ – points with $\cos\theta > 1/2$ are those with x-coordinates greater than 0.5, placing them closer to the receiver.
- **Q7:** Students may need reminding that a phasor is simply a vector whose length represents peak voltage and whose angle represents phase shift. Part (c) and (d) involve resolving into components – a good opportunity to revisit $v_x = V\cos\theta$, $v_y = V\sin\theta$.

Wrap Up – 5 Minutes

Pose any remaining questions as extensions:

- **Q8:** This is the most algebraically demanding question, requiring the compound angle formula in reverse. Students may need guidance on expanding $\sin(\phi + 25°)$ and $\sin(\phi - 20°)$, then collecting terms to find R and γ. Part (d) adds a constant damping force of 1.2 units – ensure students don't mistakenly subtract 1.2 from the minimum but rather add it to both the maximum and minimum (shifting the entire graph upward).

Remind students that they can attempt these in their own time. Encourage them to use the solutions only after attempting the problems themselves.