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Teaching Ethics in Mathematics?

You Must be Joking!

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The Oil Pipe Problem

An oil company wants to build a pipeline connecting an oil platform to a refinery (on land).

The coastline is straight; the oil platform is a distance D_1 from the coast and the refinery is D_2 from the coast.

The distance along the coastline between the platform and the refinery is H. Building the pipeline has a cost per unit length c_1 at sea, and c_2 on land.

How should the pipeline be built?

Ethics in Mathematics: An Overview

- Why do we need it?
- What is its current state?
- How do we teach it?
- Who should teach it?
- Levels of ethical engagement
- Some ‘typical’ responses
- Concluding remarks

Need for Ethics in Mathematics (EiM)

- **Both** pure and applied mathematics can lead to ethical consequences, mathematics today is an extremely sharp **double-edged sword**

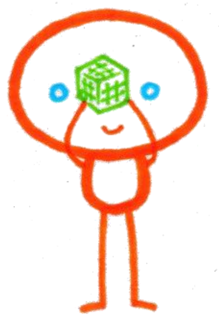
- Many examples of this such as **financial mathematics, data science, AI, statistics, mass surveillance, social networks, industrial mathematical modelling, cryptography, communication metadata, etc.**

- Mathematicians are **uniquely** responsible for the immediate **moral, ethical and legal consequences** of their work

- As educators, we train professional mathematicians, but are we really giving them a **professional training?**

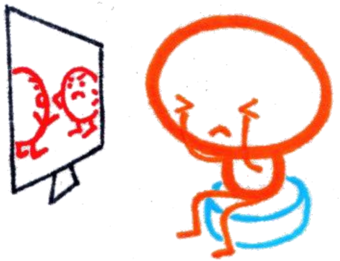
Current state of EiM: Not my problem!

This is M.



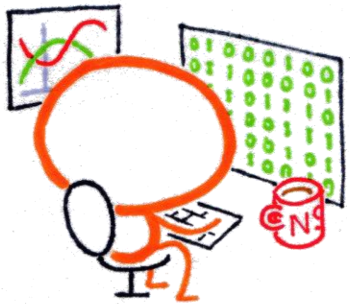
M is very clever and likes solving fun puzzles and maths problems.

M doesn't like thinking about politics or society.



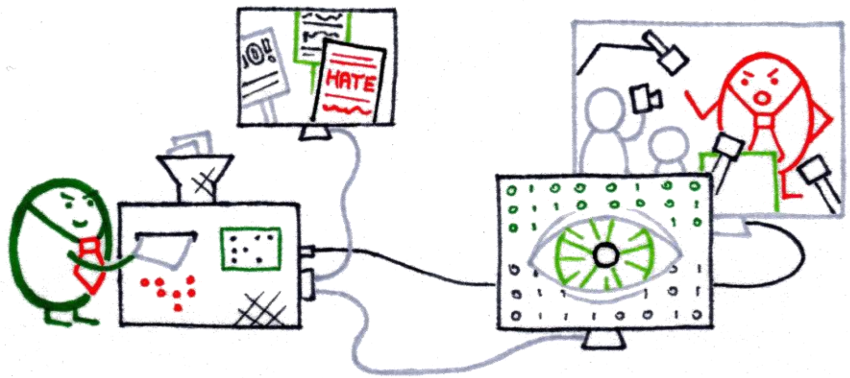
The outside world is scary.

At work, M is given shiny new problems to work out every day.



M is well looked after, and doesn't have to confront the outside world at all.

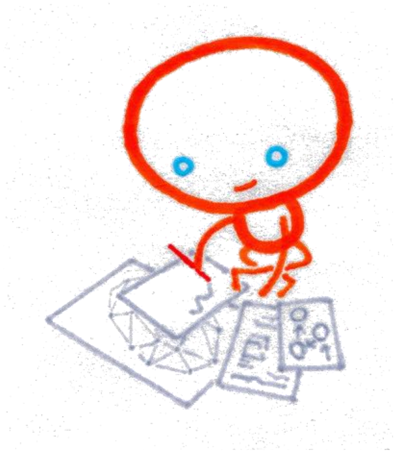
Current state of EiM: Not my problem!



And who would want to anyway?

The news seemed to be getting worse every day, which made M feel worried and scared.

Better far to ignore it all, and get back to that interesting problem…



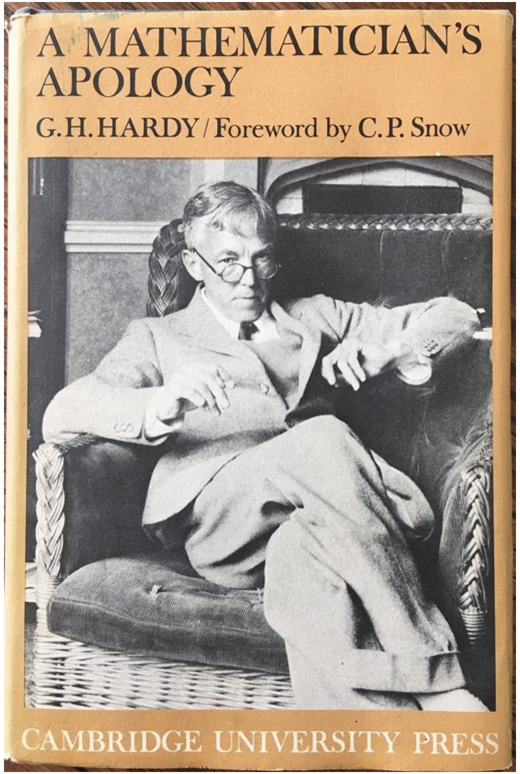
Made by Phoebe Young
www.ethics.maths.cam.ac.uk

Professional Views of EiM

- Mathematical community often still believes that while mathematics might be applied to various situations with social consequences, it is itself *free of ethical considerations*.

"No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world."

- **G.H. Hardy,** *A Mathematician's Apology* (1940)



- The *American Mathematical Society's* (AMS) ethical policy statement (https://www.ams.org/about-us/governance/policy-statements/sec-ethics) is largely **concerned only with academic ethical issues** (publishing, reviewing, discrimination)

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How can we teach EiM?

- **Interweave ethics into existing mathematics-based courses**

- involves **inserting ethical perspectives into standard mathematics courses**

(e.g. EPC's engineering ethics toolkit https://epc.ac.uk/resources/toolkit/ethics-toolkit/)

- usually done through **careful design of 'tutorial' questions** on weekly problem sheets

(will require training academic staff, as nobody has done this before)

- possibly easier for some (applied?) subjects, less so for others (pure ones?)

(not necessarily – e.g. *Cambridge Maths Tripos* questions from university's '*EiM'* project)

https://ethics.maths.cam.ac.uk/assets/teaching/Ethics_questions.pdf

Interweaving EiM – Example Problem (Applied)

Differential Equations (Sheet 4)

The evolution of an infectious disease in a population can be modelled by

$$\dot{U} = U(1 - (U + I)) - \beta U I$$
$$\dot{I} = I(1 - (U + I)) + \beta U I - \delta I$$

where U is the uninfected population, I is the infected population, $\beta > 0$ is the rate of infection, and $\delta > 0$ is the death rate caused by the disease. For $\beta = \frac{3}{4}$ determine the location and stability of the critical points of the above system in the cases:
i) $\delta = \frac{1}{5}$, ii) $\delta = \frac{2}{5}$, iii) $\delta = \frac{3}{5}$.
Thus determine the long-term outcome for the population in each case. Which of these values of δ gives the least total population in the long term? Explain why this occurs. Which do you think is worse: a disease with a mortality rate of $\frac{2}{5}$, or of $\frac{3}{5}$?

Solution

The purpose of this question is to identify what the term worse *means, and in particular,* worse *for whom? A government or hospital may prefer* $\delta = \frac{3}{5}$*; an individual being infected by the disease would almost certainly prefer* $\delta = \frac{2}{5}$*. Utilitarianism is great, unless you're on the losing end of it! Mathematicians are good at solving optimisation problems, but are seldom made to question* what *to optimise for.*

Interweaving EiM – Example Problem (Pure)

Groups (Sheet 4)

The *15-puzzle* consists of 15 small square tiles, numbered 1 to 15, which are mounted in a 4×4 frame in such a way that each tile can slide vertically or horizontally into an adjacent square (if it is not already occupied by another tile), but the tiles cannot be lifted out of the tray. A cash prize was offered for a solution to manoeuvre the tiles from the first to the second of the configurations shown below. Can you do it?
Give such a solution, or show that none exist, and outline the merits and drawbacks of offering a prize for such a puzzle.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

15	14	13	12
11	10	9	8
7	6	5	4
3	2	1	

Solution

The idea of this question is to prompt the students into asking the question "Should I be using my understanding of mathematics to trick people?" This is a fairly harmless example, but as soon as we consider such puzzles as being sold for profit, then it becomes an issue of exploiting knowledge asymmetry. Of course, any well-educated person would realise that the puzzle is impossible, but not everyone has a mathematics degree; students may not be actively aware of this.

Interweaving EiM – Example Problem (Applied)

Vector Calculus (Sheet 2)

Blood flowing in an artery is modelled as an axisymmetric flow in a cylinder of length l and radius R with axis parallel to $\hat{\mathbf{z}}$. The velocity is $\mathbf{v}(r, \theta, z) = v(r)\hat{\mathbf{z}}$, and satisfies

$$\nabla^2 v = -\frac{p}{\eta l} \ , \quad \text{with boundary conditions} \quad \left.\frac{dv}{dr}\right|_{r=0} = 0 \ , \quad v(R) = 0 \ ,$$

where p is the pressure difference between the ends of the artery, and η is the viscosity of blood. Find $v(r)$, and calculate the volumetric flow rate

$$Q = \iint_A \mathbf{v} \cdot \hat{\mathbf{z}} \, dA$$

through a cross section A of constant z. If a build-up of cholesterol reduces the radius R by a factor of $\frac{1}{5}$, by what factor does the pressure increase?
If a person's blood pressure is 50% higher than the "normal" level then they are said to be in a *hypertensive crisis* and require immediate medical attention. Compared to the radius of the artery, how thick would a layer cholesterol need to be to trigger this?

Solution

This is a modelling question, where the mathematics of model itself is quite straightforward, but the implications for the thing being modelled (in this instance, the health of a human) are incredibly serious. This is a real model, which describes the real health of real people.

Sometimes it is easy for mathematicians to become detached from the consequences of their work, through the abstract nature of mathematics. This question is designed to help students realise that their work can have real consequences and effects, in a way they can all understand and relate to.

Interweaving EiM – Example Problem (Pure)

Analysis I (Sheet 2)

Let $f_n : [0,1] \to [0,1]$ be a continuous function for each $n \in \mathbb{N}$. Find an ugly proof, and a nice proof, of the result that $h_n(x) = \max\{f_1(x), f_2(x), ..., f_n(x)\}$ is continuous on $[0,1]$ for each n. Must the function h defined by $h(x) = \sup\{f_n(x) : n \in \mathbb{N}\}$ be continuous on $[0,1]$?

Solution

The purpose of this question is to prompt the students to think about, and realise, that mathematics is viewed beyond "correct vs incorrect"; we have our own stylistic and professional norms and beliefs on what is "good" mathematics, and what is "bad" mathematics.

How can we teach EiM?

- **Develop standalone courses on ethics in mathematics**

- already exist in disciplines such as engineering, computer science, law etc., **but very rarely in mathematics** (particularly in the UK)

*“If you’d asked me 20 years ago whether mathematicians needed to be exposed to ideas of ethics, I would have said ‘Clearly, that is irrelevant to mathematicians. Now I really think that this is something we have to think about, in the same way that **engineers have courses looking at ‘What it means to be a professional engineer’, and ‘Ethics, and your responsibilities as an engineer’**. I think that is something that we have to think about as mathematicians now.”*

- ***Prof. Michael Giles,*** *Head of Applied Mathematics, University of Oxford* (2018)

EiM project at Cambridge

- **Informal, non-examinable 8-lecture EiM course** taught to Cambridge mathematics Tripos students by Dr. Maurice Chiodo entitled *"Ethics for the Working Mathematician"*

https://ethics.maths.cam.ac.uk/course/lectures/

- Course based on the following rationale as a ***'proof for ethics':***

Assumption: students will not initially and intuitively accept the premise that there are ethical issues in mathematics

Two-fold proof: a) **There exist** some ethical issues in mathematics

b) **For all** mathematics that we do, there are ethical issues

EiM project at Cambridge

- Course involved 20 contact hours comprising **lectures, interactive case-study exercises and challenging discussions (https://ethics.maths.cam.ac.uk/cases/)**
- Content included a total of 8 topics covered:
  - *Week 1: Introduction to EiM*
  - *Week 2: Financial mathematical modelling*
  - *Week 3: Cryptography, surveillance and privacy*
  - *Week 4: Fairness and impartiality in algorithms and AI*
  - *Week 5: Regulation, accountability and law*
  - *Week 6: Understanding the behaviour of the mathematical community*
  - *Week 7: How to survive as a mathematician at work?*
  - *Week 8: Looking into the future, what more can mathematicians do?*
- Featured no assessment, though **mathematical problems with ethical components** woven into other courses (as discussed)

Who should teach EiM?

- As argued by Chiodo and Bursill-Hall (2019), EiM courses **should not be delivered by philosophers or ethicists** (unlike ethics courses in other disciplines, mathematics students are not intrinsically aware of ethical issues)

- In fact**, these should be taught by mathematicians themselves** (even if not professionally trained ethicists, just as linear algebra lecturers are not always experts in algebra)

- For instance, **history of mathematics courses** (e.g. at Oxford, Cambridge) **are taught formally by mathematicians, not historians**

- EiM too requires an **authentic, contextual, discipline-specific** form of teaching

Levels of ethical engagement

- Based on their observations, Chiodo and Bursill-Hall (2018) formulated **four levels of ethical engagement** and a **non-level:**

  - **Level 0:** Believing there is no ethics in mathematics
  - **Level 1:** Realising there are ethical issues inherent in mathematics
  - **Level 2:** Speaking out to other mathematicians
  - **Level 3:** Taking a seat at the tables of power
  - **Level 4:** Calling out the bad mathematics of others

- Majority of mathematicians are at **Level 0** with the mindset:

"NO-ONE needs to think about what ANYONE is doing."

Some ‘typical’ responses

- **“Ethics is just an opinion; it is not mathematically precise”**

Yes, it is subjective, but still important and useful (medical, legal, professional ethics in other disciplines are also not absolute, but does not mean it should not be considered)

- **“This is not my job”**

Yes, it is, then whose job is it? (management/legislators do not have the skills to interpret)

- **“I see no other mathematicians doing this, so why should I?”**

Just because not seen anyone do it, does not mean it should be done (original research?)

- **“What qualifies a non-philosopher to speak about ethics in mathematics?”**

no existing qualifications in ethics, we do not rely philosophers to lay out medical, legal or engineering ethics, doctors speak about ethics, why not mathematicians?

Concluding remarks

- **Faculty support is therefore critical** for setting up EiM courses, but this can be hard (responses such as "*we're a mathematics department, why are we teaching ethics?"*)

- Ethics *is* a matter of opinion, but that **does not mean it cannot be addressed**

- When other disciplines face ethical issues and train professionals to deal with them, **how can we exclude ourselves from them?**

- As educators, we have a **duty to teach our students** how to use the mathematics they learn responsibly

References

A large majority of references can be found from the Cambridge EiM project website https://ethics.maths.cam.ac.uk/publications/

- The role of ethics in a mathematical education, M. Chiodo, R. Vyas, Ethics in Mathematics Discussion Papers, 2019
- Teaching Ethics in Mathematics, M. Chiodo, P. Bursill-Hall, LMS Newsletter **485**, 22-26, November 2019. Republished in the EMS Newsletter **114**, 38-41, December
- Four Levels of Ethical Engagement, M. Chiodo, P. Bursill-Hall, Ethics in Mathematics Discussion Papers, 2018

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Thank you for your time!

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