Queen Mary University of London

Problem 1 Solution – Pipeline Construction

Problem 1 - Pipeline Construction
Mathematical topic: Optimisation
Contribution to SDGs : Affordable and Clean energy (SDG 7), Industry, Innovation and Infrastructure (SDG 9), Life below water (SDG 14), Life on land (SDG 15)

The cost-minimisng path (assuming there are no other associated costs) is calculated by Snell's law and trigonometry.
This involves creating a diagonal straight line under water from the platform to the shoreline and finally a straight line down the shoreline.

Calculations:
Let x be the length of the pipe along the shoreline. The under-water diagonal pipe (L), is computed by Pythagoras' Theorem.

$$L = \sqrt{D_1^2 + (D_2 - x)^2}$$

Thus the total cost of the pipe in terms of x, $C(x)$, is given by

$$C(x) = c_2 x + c_1 \sqrt{D_1^2 + (D_2 - x)^2}$$

Differentiating with respect to x, and looking for a minima, we get

$$0 = \frac{dC}{dx} = c_2 - \frac{c_1(D_2 - x)}{\sqrt{D_1^2 + (D_2 - x)^2}}$$

And thus

$$\frac{c_2}{c_1} = \frac{(D_2 - x)}{\sqrt{D_1^2 + (D_2 - x)^2}}$$

Let θ be the angle the underwater pipe makes with the shoreline. Observe that we have

$$\cos(\theta) = \frac{(D_2 - x)}{\sqrt{D_1^2 + (D_2 - x)^2}} = \frac{c_2}{c_1}$$

Thus

$$\frac{D_1}{D_2 - x} = \tan(\theta) = \tan(\arccos(\frac{c_2}{c_1}))$$

Rearranging gives

$$x = D_2 - \frac{D_1}{\tan(\arccos(\frac{c_2}{c_1}))}$$

Hence, the minimum cost pipe has 2 segments:

1. **Segment 1** has a straight pipe on the shore that extends up to a distance x from the refinery.
2. **Segment 2** has a straight pipe beneath water, that connects the end of segment 1 pipe to the platform.

Challenges of optimisation: Economic actions involves externalities such as:

- **Environment damage** – pipe might go through a coral reef or protected habitats.
- **Disruptions to existing infrastructure** – pipe might go through a school.
- **Maintenance challenges** – for example, leaks and rusts.

Solution – Policymakers must have a holistic view of these effects and mathematicians should clearly communicate the assumptions and simplifications of their models.

Problem 2 Solution – Mercury Contamination

Problem 2: Mercury Contamination
Mathematical topic: Differential equations
Contribution to SDGs : Clean Water and Sanitation (SDG 6), Responsible Consumption and Production (SDG 12)

A chemical accident took place near a small village in Peru. Therefore, this problem is designed to demonstrate students how mathematics can be used to model local environmental disasters.

Calculations:
Let the region's local water reservoir have a volume : V
The inflow and outflow of the reservoir have a flow rate: r.
Amount of mercury in the reservoir at time (t) is: $x(t)$.

Assumption: reservoir was clean at the beginning i.e., $x(0) = 0$. Thus, the concentration of mercury flowing into the reservoir is $C(x)$.

Consider "rate of change = rate of chemical inflow - rate of chemical outflow".

$$\frac{dx}{dt} = rC_e - r\frac{x}{V}$$

$$\frac{dx}{dt} + r\frac{x}{V} = rC_e$$

Let $C = \frac{x}{V}$ be the concentration of [chemical] in the reservoir. Plugging in we obtain a differential equation purely involving concentrations.

$$V\frac{dC}{dt} + \frac{r}{V}C = \frac{r}{V}C_e$$

To solve this, multiply with the integrating factor $e^{\frac{r}{V}t}$ to obtain

$$e^{\frac{r}{V}t}\frac{dC}{dt} + \frac{r}{V}Ce^{\frac{r}{V}t} = \frac{r}{V}C_e e^{\frac{r}{V}t}$$

$$\Rightarrow \frac{d}{dt}\left(\frac{r}{V}Ce^{\frac{r}{V}t}\right) = \frac{r}{V}C_e e^{\frac{r}{V}t}$$

$$\Rightarrow \frac{r}{V}Ce^{\frac{r}{V}t} = \int \frac{r}{V}C_e e^{\frac{r}{V}t}dt + const$$

$$\Rightarrow C(t) = \frac{V}{r}e^{-\frac{r}{V}t}\int \frac{r}{V}C_e e^{\frac{r}{V}t}dt + e^{-\frac{r}{V}t} \cdot const$$

Plug in the initial condition $x(0) = 0 \Rightarrow C(0) = 0$ to get

$$C(t) = \frac{V}{r}e^{-\frac{r}{V}t}\int \frac{r}{V}C_e e^{\frac{r}{V}t}dt - \frac{V}{r}e^{-\frac{r}{V}t}$$

Key questions to think when calculating the solution of this problem:

- Will the **pollution** of the reservoir ever reach a **dangerous level** in the reservoir ?
- What is deemed a **"safe" level** of mercury in the reservoir ?
- How closely does the **concentration** of the reservoir follow the inflow of pollutant chemicals?
- Will the reservoir reach an **equilibrium concentration** of mercury ?

Why are these questions important ?

Mercury is **poisonous** so drinking an extensive amount is highly dangerous. This solution is an **estimation** of when the water is safe to drink, thus this links with **safety prediction**. On top of that, this helps governments to plan things such as how much water from alternate source they need to obtain.

Problem 3 Solution- Simpson's Paradox

Problem 3 – Simpson's paradox
Mathematical topic : Probability
Contribution to SDGs : Gender Equality (SDG 5), Reduced Inequalities (SDG 10)

This problem demonstrates **Simpson's paradox**, a statistical phenomena in which a trend appears in several groups of data but disappears or reverses when the groups are combined. It also highlights gender disparities in mathematics admissions, enhancing the understanding of systematic inequalities.

Calculations:
The success rates for male and female applicants based on preference (Applied or Pure Mathematics) and overall are calculates as follows:

	Prefer applied	Prefer pure	Total
Female	$\frac{18}{270} = \frac{14}{210}$	$\frac{12}{30} = \frac{4}{10}$	$\frac{30}{300} = \frac{10}{100}$
Male	$\frac{15}{350} = \frac{9}{210}$	$\frac{195}{650} = \frac{3}{10}$	$\frac{210}{1000} = \frac{21}{100}$

Observations:
Simpson's paradox is observed here: Females have higher success rates within each sub departments, yet their overall acceptance rate is lower than males; **0.21 (male)** vs **0.1 (female)**.

Explanation:
- The largest male cohort (those who prefer pure mathematics – 650 applicants) has a high success rate of 0.3, raising the overall male success rate.
- However, the largest female cohort (those who prefer applied mathematics – 270 applicants) has a much lower success rate of 0.067, which drags down the overall female success rate.

This phenomena demonstrates the importance of examining sub-groups dynamics when analysing data to avoid misinterpretations.

Key questions to think about**:**
1. Analysing fairness in data aggregation:
- How do different cohort sizes influence the outcomes?
- What is the impact of Simpson's paradox on policy making in education?

2. Understanding sustainable systems
- How can mathematics be used to promote equity and fairness in admission policies?
- What steps can institutions take to ensure diversity and inclusivity in male dominant fields?

3. Sustainability in decision making
- How does this analysis connect with broader sustainability goals, such as reducing gender equality **(SDG 5)** and ensuring quality education **(SDG 4)**.

Why these questions matter:
- Mathematical analysis supports sustainable problem solving by revealing hidden trends like Simpson's Paradox.
- In this way, it is possible to avoid misleading conclusions and promote the development of more inclusive, fair outcomes.
- These insights enable educators, policymakers, and researches to incorporate equity and sustainability into their decisions, supporting diversity and sustainable development.