Queen Mary University of London

Examples of mathematical problems with embedded sustainability

Problem 1 - Pipeline construction

Mathematical topic: Optimisation

Contribution to SDGs : Affordable and Clean energy (SDG 7), Industry, Innovation and Infrastructure (SDG 9), Life below water (SDG 14), Life on land (SDG 15)

Context of the problem: An oil company plans to build a pipeline from an offshore platform D_1=13 km from the coast to a refinery D_2 = 10 km along the coastline from the nearest point onshore. The cost is c_1= £90,000 per km for underwater construction and c_2 = £60,000 per km on land.

- Calculate the optimal pipeline length x.
- What factors should be considered when determining the most cost-effective route?

Mathematical approach : This problem focuses on finding the optimal length need to minimise the total construction cost. By applying the **Pythagorean theorem with trigonometry and differentiation from calculus**, we can determine the optimal length 'x' along the coast where the pipeline should transition from underwater to land. This involves setting up a cost function $C(x)$, which can be minimised to find the length 'x'.

Key sustainability insight: Cost minimisation is the primary mathematical focus, but the real-world implications of this problem extend beyond mere economics.

- **Environmental impact:** The pipeline's route might go through delicate marine areas like coral reefs and impact wildlife and ecosystems on land. Building pipelines in these locations can cause lasting environmental damage.
- **Human impact:** On land, the pipeline could disrupt local communities, cultural heritage sites, or existing infrastructure.

Therefore, mathematical models should include externalities such as environmental harm, damage to existing infrastructure, and societal costs. This encourages engineers to adopt a holistic approach instead of focusing only cost optimisation.

Problem 2: Mercury Contamination

Mathematical topic: Differential equations

Contribution to SDGs: Clean Water and Sanitation (SDG 6), Responsible Consumption and Production (SDG 12)

Context of the problem: A chemical accident occurred near a small village in Peru, contaminating the local water reservoir, which has a volume V. The inflow and outflow rates are represented by r, and $x(t)$ denotes the amount of mercury in the reservoir over time, starting with $x(0)$ = 0 (clean water). The concentration of mercury entering the reservoir is $C_e(t)$.

- Set up and solve a differential equation that describes the concentration of mercury in the reservoir.
- What are some relevant questions you can ask about the concentration of mercury in the reservoir? How much does it matter?

Mathematical approach: The model for the buildup of mercury in the reservoir can be set up using **differential equations**, in which mercury inflow depends on the concentration of pollutants entering over time $C_e(t)$. Setting up an equation for the rate of change of mercury $x(t)$, allows us to predict its concentration over time. This can also be extended to account for repeated pollution events, such as illegal dumping, and helps determine when mercury, levels might exceed safe limits.

Key sustainability insight: This mathematical model allows us to:

- Estimate the **time at which the reservoir will become unsafe** for use due to high mercury concentration level.
- Determine **how long it will take for the water to become safe** again after pollution stops, considering uncertainties such as measurement errors or even mercury distribution.
- Evaluate potential environmental risks, ensuring **responsible consumption and production** of resources in line with SDG 12.

Such a model offers key insight for managing environment disasters, but students should recognise that mathematics is only part of the solution. The incorporation of real-world factors are essential to confirm water safety, highlighting the important of careful decision-making.

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Problem 3 – Simpson's Paradox
Mathematical topic : Probability
Contribution to SDGs : Gender Equality (SDG 5), Reduced Inequalities (SDG 10)

Context of the problem: In a particular admissions cycle, a mathematics department observes a higher success rate for male applicants than for female applicants. To investigate whether this is the same across the two sub-departments of Pure and Applied Mathematics, the following year the department asks each applicant to give their preference for pure or applied mathematics (they are not allowed to be ambivalent) and records the resulting statistics as shown:

Total:

	Applications	Successful
Female	300	30
Male	1000	210

Prefer applied:

	Applications	Successful
Female	270	18
Male	350	15

Prefer pure:

	Applications	Successful
Female	30	12
Male	650	195

- Compare the success rates for male and female applicants that prefer applied mathematics, prefer pure mathematics and their success rates overall.
- What do you notice? Why is this possible? This is known as **Simpson's paradox**.

Mathematical approach: This problem focuses on applying **probability** and **data analysis** to demonstrate Simpson's paradox, a statistical phenomena in which a trend appears in several groups of data but disappears or reverses when the groups are combined. The question involves calculating and comparing the success rates for male and female applicants within each of the subgroups, which are Pure Mathematics and Applied mathematics, as well as for the whole group. By examining how group sizes and success rates contribute to combined data, the analysis provide a deeper understanding of the mechanics of the paradox.

Key sustainability insight: This question combines mathematics with real-world issues related to gender equality (SDG 5) in education and professional environments.

- **Gender disparities in STEM**: This question draws attention to gender disparities in mathematics and other STEM fields worldwide.
- **Broader inequalities**: It highlights how misinterpreting data can reinforce existing inequalities and shows the importance of transparency and accountability in data analysis, aligning with SDG 10.

This issue demonstrates the crucial role of mathematics in undercovering hidden disparities and supporting informed decision-making to promote fairness and equity.

Explore the *'Making Diversity Count'* research project
The research project highlights the importance of representation in STEM, linking to our discussion of gender equality (SDG 5) and equity (SDG 10).

Discover biographical posters of STEM Champions
Learn about diverse mathematician and scientists who challenged stereotypes and inspire the next generation!