Sustainability in Mathematics Problems? You Must Be Joking!

Rehan Shah MIMA, Queen Mary University of London

Motivation

Despite the prevalence of the UN Sustainable Development Goals (SDGs) [1] since 2015, several reports and studies [2–4] have noted that the incorporation of sustainability within universities finds the greatest barriers within teaching, with curricula often failing to address key environmental and ethical issues. Within the mathematical sciences, a report by the American Mathematical Society [5] also recommended that sustainability in mathematics should permeate education curricula for high-school and university students. This situation reflects the need for academics and educators in higher education to develop a toolkit of resource materials that can serve as a reference guide for the effective and systematic integration of sustainability into university mathematics curricula.

At policy level too, there is a growing consensus for integrating sustainability principles and practices into the curriculum and pedagogy of mathematics programmes. Learning about the environmental, social and economic dimensions of sustainable development equips students with the knowledge and skills needed to address sustainability challenges in their professional practice. For example, the *Mathematics, Statistics and Operational Research (MSOR) Subject Benchmark Statement* [6], published by the Quality Assurance Agency for Higher Education in March 2023, explicitly highlights the need for mathematical science degrees to include technical problems where the motivation or context of the question stems from a sustainability issue.

Basic principles for embedding sustainability

In this section, I include some best-practice principles that I have used when integrating sustainability into some of my university-level mathematical problems and exercises.

Interweaving into existing course materials

Integrating sustainability within STEM courses is 'largely a matter of providing context for what is already being taught', context that also makes the material already being taught seem 'more relevant' [7, p. 717]. The technique of 'micro-insertion', as described in the context of embedding ethics, can also be succinctly adapted for incorporating sustainability issues, in that these 'are not add-ons; they work like an alloy, adding strength to the course without adding volume' [7, p. 724], i.e. they need not be perceived as components to be introduced in lieu of existing technical material.

Integration in a seamless and organic manner

The inclusion of sustainability aspects into mathematical problems does require a great deal of care, thought and gradual experience, as it ought to be done in a seamless and organic manner, without appearing to be artificial or contrived. The focus should be on how students engage with these themes, rather than merely introducing sustainability as an add-on component [8]. Often, this can be avoided by ensuring that such concepts emerge

Figure 1: UN Sustainable Development Goals [1]

naturally from the technical nature of the mathematics of the problem itself, i.e. through an enlargement of the context of a problem by aligning it to a realistic scenario (see example problems below).

Subjective and reflective aspects

The sustainability elements of a question will often involve open-ended discussions and subjective aspects, prompting deeper reflection from both students and instructors [9]. This is unlike the objective *right or wrong* answers provided to the technical parts of the problem. Therefore, even the guided *solution sketches* (as shown in the example problems) should not be treated as exact and fixed in nature and may be altered, tweaked or modified on subsequent usage.

Links with the UN Sustainable Development Goals

By establishing direct links between the overarching themes emerging from the technical problems and mapping them to key sustainability concepts within the 17 SDGs (Figure 1), formulated by the United Nations as part of its 2030 Agenda for Sustainable Development, students will be able to acquire a much more holistic view of sustainability within the context of their technical learning in STEM disciplines [10].

Examples of mathematical problems with embedded sustainability

The following three example problems (from [11], CCBY-SA 4.0, with minor adaptations) illustrate ways in which some of the basic principles outlined earlier have been used to incorporate sustainability aspects within technical exercise questions piloted in first- and second-year undergraduate mathematics modules. Partial solution comments have been included here for brevity (focusing primarily on the sustainability elements). Interested readers can refer to [11] for full details of the mathematics.

Problem 1: Pipeline construction

Topic: Optimisation (from [11, p. 56–57])
SDG Mapping: 7 (Affordable and Clean Energy), 9 (Industry, Innovation and Infrastructure), 14 (Life Below Water) and 15 (Life on Land)

An oil company wants to build a pipeline connecting an oil platform to a refinery (on land). The coastline is straight. The oil platform is at a distance of D_1 from the coast. The refinery is on the coastline, a distance D_2 from the point on the coast closest to the platform. Building the pipeline incurs costs per unit length of c_1 at sea and c_2 on land.

(a) Calculate the optimal length of the pipeline.

(b) What factors need to be considered when providing a response to this question?

Solution comments
The cost-minimising path (assuming that there are no other associated costs) is a straightforward exercise in trigonometry and calculus, which yields the total cost of the pipe $C(x)$ in terms of pipe length x along the shoreline:

$$C(x) = c_2 x + c_1\sqrt{D_1^2 + (D_2 - x)^2},$$

which when minimised with respect to x gives

$$x = D_2 - \frac{D_1}{\tan(\cos^{-1}(c_2/c_1))}.$$

But who said we were optimising over cost? This is an assumption often ingrained into mathematicians while they are students, but it need not always be the right way to optimise.

Economic actions almost always have externalities, such as possible damage to the environment (the pipe may go through a coral reef or protected habitat) or to existing infrastructure (it may go through a school or a site of archaeological significance).

How can we mathematically model the environmental and human impacts of laying this pipe? There are numerous factors to consider, and students, much like policymakers, should take a holistic view of these effects and at least be aware of the implications of basing decisions solely on economic factors.

Problem 2: Environmental disasters

Topic: Differential equations (from [11, p. 44–45])
SDG Mapping: 6 (Clean Water and Sanitation) and 12 (Responsible Consumption and Production)

There was a chemical accident near a small village in Peru. The region's local water reservoir has a volume V. The inflow and outflow of the reservoir depend on the flow rate r. Let $x(t)$ be the amount of mercury in the reservoir at time t. Assume that the reservoir was clean at the beginning, i.e. $x(0) = 0$. Let $C_e(t)$ be the concentration of mercury flowing into the reservoir.

(a) Set up and solve a differential equation describing the concentration of mercury in the reservoir.

(b) What are some relevant questions you can ask about the concentration of mercury in the reservoir? How much does it matter?

Solution comments
This question is designed to show students that very simple mathematics can be used to model local environmental disasters. It could be used as an example of how the environment is being unsustainably treated.

For part (a), students will consider that rate of change = rate of chemical inflow − chemical outflow and set up the model as

$$\frac{\mathrm{d}x}{\mathrm{d}t} = rC_e - r\frac{x}{V},$$

from which they can transform the equation by setting and solving for the concentration of mercury in the reservoir $C(t) = x(t)/V$ after using the given initial condition.

For part (b), possible questions for students to consider can include:

- Will the pollution of the reservoir ever reach a dangerous level?
- What is deemed a 'safe' level of mercury in the reservoir?
- How closely does the concentration of the reservoir follow the inflow of pollutant chemicals?
- Will the reservoir reach an equilibrium concentration of mercury?

For the sub-question 'How much does it matter?', students should identify that we are dealing with poison in drinking water, so it matters immensely!

Problem 3: Simpson's paradox

Topic: Probability (from [11, p. 68–69])
SDG Mapping: 5 (Gender Equality), 10 (Reduced Inequalities)

In a particular admissions cycle, a mathematics department observes a higher success rate for male applicants than for female applicants. To investigate whether this is the same across the two sub-departments of pure and applied mathematics, the following year the department asks each applicant to give their preference for pure or applied mathematics (they are not allowed to be ambivalent) and records the resulting statistics (see Table 1).

Table 1: Admission statistics for mathematics [11, p. 68].

	Total	
	Applications	**Successful**
Female	300	30
Male	1000	210

	Prefer applied	
	Applications	**Successful**
Female	270	18
Male	350	15

	Prefer pure	
	Applications	**Successful**
Female	30	12
Male	650	195

(a) Compare the success rates for male and female applicants who prefer applied mathematics or prefer pure mathematics with their success rates overall.

(b) What do you notice? Why is this possible?

Solution comments

The purpose of this question is to demonstrate Simpson's paradox in which a trend appears in several different groups of data but disappears or reverses when these groups are combined. It also attempts to highlight the immense gender disparity in many mathematics departments around the world.

For part (a), students will easily compute the fraction of students successful in the different categories (see Table 2).

Table 2: Proportion of successful applicants, illustrating Simpson's paradox [11, p. 68].

	Prefer applied	**Prefer pure**	**Total**
Female	$\frac{18}{270} = \frac{14}{210}$	$\frac{12}{30} = \frac{4}{10}$	$\frac{30}{300} = \frac{10}{100}$
Male	$\frac{15}{350} = \frac{9}{210}$	$\frac{195}{650} = \frac{3}{10}$	$\frac{210}{1000} = \frac{21}{100}$

For part (b), it is evident from the calculations in Table 2 that female applicants with a given preference (pure or applied mathematics) have a higher success rate, but have lower overall success rates, than male applicants. This is Simpson's paradox.

The heuristic reason for why this is possible is that the largest male cohort (those who prefer pure) has a much higher acceptance rate than the largest female cohort (those who prefer applied). So the overall acceptance of men is dominated by those who prefer pure, while the overall acceptance of women is dominated by those who prefer applied. This is a great lesson for showing students why it is usually a terrible idea to take averages of averages.

Conclusion

I hope that this article will provide academics and educators with some insight into how sustainability concepts can be integrated into technical, mathematical problems prevalent within STEM curricula. This may help to motivate lecturers to design their own versions of similar exercises, in response to ongoing calls to enhance the restructuring of our university programmes to better prepare future STEM graduates for tackling global sustainability challenges.

The inspiration for the title arises from the reaction I often receive from academic colleagues encountering this topic for the first time. It is similar to the title of physicist Richard Feynman's book *'Surely, You're Joking Mr. Feynman!'* [12], which was based on a comment made when Feynman, then a postgraduate student, asked for both cream and lemon in his tea!

References

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