Queen Mary University of London

Examples of mathematical problems with embedded ethics

Problem 1 - Confronting Your Boss with Logic

Topic: Propositional logic

Context of the problem: Your boss has given you a task. The task is well within your technical capability, but you are not sure whether it would be legal or ethical. You feel uneasy, but your boss tells you: '*If you don't do it, then someone else will.*'

Do you think that the boss' argument is cogent? Does that depend on who you are or what the project is? How would you answer your boss?

Mathematical approach : This problem can be mathematically examined by investigating the **reasoning** and **logic** behind the boss's statement.

- The statement *'If you don't do it, then someone else will,'* shows that someone else could replace you to do the task.
- Therefore, by using the tools of **propositional logic**, you can work out the likelihood for someone else to do the task instead of you.

Key ethical insight: Ethical decision making considers both logical reasoning and personal values.
Interpersonal integrity: It is essential to act morally as unethical behaviour could lead to unreliability.
Responsibility: Promoting ethical decision-making in the workplace can result in a positive environment to work.

This exercise allows students to delve into logical reasoning within the field of boolean algebra.

Problem 2: Ethics of Crime Scene Investigation

Topic: Mathematical modelling

Context of the problem: A detective arrived at the scene of a crime at 5:00pm, finds a warm cup of tea and measures its temperature at 40°C. By 5:30pm the tea's temperature has dropped to 30 °C.

a) The police approach you with this data and ask you when the tea was likely made. **Briefly discuss any questions that you still need to ask** the police officers and their potential ethical relevance. **What are the potential barriers to communication?**

a) The police are unable to provide you with more information, but ask you to give an estimate based on idealised conditions and a constant room temperature of 20°C. Giving all mathematical details and assumptions, **use Newton's law of cooling to estimate when the tea was likely made. What error margins might your computation have, and what might the consequences of these be?**

Your task is to **identify** questions the police could ask to estimate when the tea was made and to investigate potential errors.

Mathematical approach: This problem can be solved by applying **Newton's law of cooling**, which explains how the temperature of an object varies over time. Therefore, this can be solved by using **differential equations**, where:
T(t) is the **temperature** of the tea at time t and the **room temperature (constant)** is 20°C (T_e).

$$\text{Thus: } \frac{dT}{dt} = k(T_e - T)$$

Key ethical insight: The mathematical solution does not provide a whole picture as it solely relies on idealised assumptions such as the room temperature is constant when in reality, the room's temperature may not have been constant.

This exercise will demonstrate students how to apply differential equations in real-world problems.

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Problem 3 – Mathematics of Military Engagement

Topic: Differential equations

Context of the problem: Two armies, R (red) and B (blue), have soldier counts $m_R(t)$ and $m_B(t)$, respectively.

1. Losses of each army are proportional to the strength of the opposite army, with proportionality constants a_R and a_B. **Set up a system of differential equations to describe the strengths of both armies over time.**

2. **Derive Lanchester's square law**: $m_R^2(t) - m_R^2(0) = a_B(m_B^2(t) - m_B^2(0))$

3. How would you **explain a_R and a_B** in everyday language? **Would you feel comfortable solving such problems** in your career?

Mathematical approach : A system of differential equations is used to describe the dynamics of two armies, R (red) and B (blue), where losses are proportional to the strength of the opposing army.

- **Lanchester's Model** – By using this model, differential equations are derived to predict how the number of soldiers in each army changes over time and how this affects the outcome of the engagement.

The coefficients a_R and a_B represent the 'killing rate' for each side, influencing the strategy and likely outcomes of the battle.

Key ethical insights: This question highlights the real-world consequences of using mathematics to model military engagements and warfare. Some important ethical considerations include:

- **The impact of mathematical models on human life** : Abstract mathematics, can directly influence decisions in warfare, underscoring the profound ethical responsibility in applying such models.
- **Moral implications of participations** : Understanding proportionality constants raises significance ethical concerns, prompting reflection on whether contributing to such models aligns with one's personal values and professional ethics.

Therefore, it is crucial for mathematicians to critically consider the moral implications of their work.

Problem 4 – Mathematical Communication

Topic: Numerical methods

Context of the problem:

(a) A smooth function $F : [a, b] \to \mathbb{R}$ satisfies $F(a) < 0 < F(b)$ and $F'(x) > 0$, ensuring a unique solution c to $F(x) = 0$. Numerical approximations to c can be obtained using the Newton-Raphson iteration method, which converges quadratically. **State the Newton-Raphson formula and explain it graphically for a none technical audience.**

(b) Finite Difference methods (forward, backward, and central) are used to approximate the derivative F' (a) with step size h. **State the formulas for these methods and explain them graphically for a non-technical audience.**

Mathematical approach: This problem involves using two mathematical techniques:

1. **Newton-Raphson iteration**: This method numerically approximates the root c of a function $F(x) = 0$. By iteratively drawing tangents to the curve $F(x)$ and finding where they intersect the x-axis, we create successively better approximations of c. This method converges quadratically, making it efficient for practical applications.
2. **Finite difference methods**: These methods estimates the derivative $F'(a)$ of a function using discrete data points and a step size h. Forward, backward, and central differences approximate the slope (or rate of change) at a point a, with central differences offering higher accuracy.

Both techniques are visualised graphically to help communicate complex ideas in an accessible manner.

Key ethical insight: This problem demonstrates the importance of effective mathematical communication in real-world contexts, particularly to non-technical audiences. Key ethical insights include:

- **Accessibility and inclusivity:** Simplifying mathematical concepts ensures broader understanding, empowering non experts to engage in informed discussions.
- **Transparency and responsibility:** Clear and accurate explanations build trust, ensuring stakeholders understand the limitations and implications of mathematical models without risking misinterpretation.

Therefore, mathematical communication should prioritise clarity and inclusivity to promote ethical understanding and responsible decision making across diverse audiences.