Problem 1 Solution – Confronting Your Boss with Logic

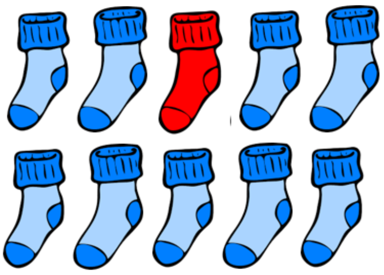
<u>Problem 1 – Confronting Your Boss with Logic</u>
Topic: Propositional logic

Your boss has given you a task, however, you are unsure if it is legal or ethical. You feel uneasy, but your boss insists that ***'If you don't do it, someone else will.'*** Is this a valid question? Should you accept this reasoning?

This is a **flawed assumption**:

- The phrase assumes at least one person will carry out the task.
- This is not true as if no one else is available or willing to do it, the task may not happen at all.

Sock Analogy – to understand the problem



- Imagine a bag containing **1 red** sock and **9 blue** socks. There are **10 people** who pick a sock from the bag. The statement ***'If you don't pick the red sock, someone else will'*** is only **true if all 10 people pick a sock**.
- If **not all 10 people participate**, there is a chance that the **red sock may never be picked**. What is true instead is ***'If you don't pick the red sock, someone else might.'***

This shows that just because an action can happen it does not mean it will happen. The probability of an event happening depends on how many people are actually willing to do it.

Therefore, going back to the problem:

- The correct phrasing should be ***'If you don't do it, someone else might.'*** So now we are accounting for the probabilities of the event's occurrence.
- If **many people can and want to do the task**, then it has **a higher probability** of occurring.
- However, if **few people** are capable of doing the task, then its **probability will be low**.

<u>Ethical Considerations:</u>

- Ethical decision making depends on evaluating its consequences not just its probability of occurrence.
- Justifying actions based on the belief that something will happen can lead to unethical decisions since it ignores the chance that it may not occur.

<u>Key Questions to Consider:</u>

- Does the assumed probability justify the ethical responsibility associated with this problem?
- If something unethical is presented as unavoidable, does that make it right?

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Problem 2 – Ethics of Crime Scene Investigation
Topic: Mathematical modelling

This question emphasises the importance of assumptions in mathematical models and the challenges of implementing them in real-world situations. The problem shows that assumptions are not always accurate and can lead to ethical issues. Therefore, this highlights the significance of critical thinking and clear communication.

Solution to **part (a):** Students should realise

- Room temperature is unknown, and varying temperature affects the accuracy of the estimate.
- Factors such as movement and drinking can alter the tea's cooling rate.
- Police may lack mathematical training; therefore, they should communicate clearly.
- Assumptions without evaluation can lead to flawed conclusions.

Solution to **part (b)**: Using **Newton's Law of Cooling**, the temperature of an object changes at a rate proportional to the difference between its temperature and the ambient temperature.

Information from the question:

- $T_e(t)$ is the unknown temperature of the surrounding room.
- $T(t)$ is the **temperature** of the tea at time t.
- Assume **room temperature** T_e **(constant)** = **20°C**

Therefore: $$\frac{dT}{dt} = k(T_e - T) \quad (1)$$

Using the boundary conditions:

- At $t = 0$ (5:00pm), $T(0) = 40$°C
- At $t = 1$ (5.30pm), $T(1) = 30$°C

This leads us to solve the first-order ordinary differential equation:

$$\frac{dT}{dt} = k(20 - T) \quad (2) \longrightarrow \frac{dT}{dt} + kT = 20k \quad (3)$$

Set: $\mathbf{u} = e^{kt}$

$$u\frac{dT}{dt} + T\frac{du}{dt} = 20\,uk \quad (4)$$

$$e^{kt}\,T = 20e^{kt} + c \quad (5)$$

$$T(t) = 20 + ce^{-kt} \quad (6)$$

Now **use the given temperature constraints to obtain the constant c and k:**

$$T(0) = 20 + c = 40 \longrightarrow c = 20 \quad (7)$$

$$T(1) = 20 + 20e^{-k} = 30 \longrightarrow k = -\ln\left(\frac{1}{2}\right) \quad (8)$$

Thus, the **temperature at time t is**:

$$T(t) = 20 + 20e^{-\ln\left(\frac{1}{2}\right)t} \quad (9)$$

Solve for t:

$$100 = 20 + 20e^{\ln\left(\frac{1}{2}\right)t} \quad (10)$$

$$t = \frac{\ln(4)}{\ln\left(\frac{1}{2}\right)} = -2 \quad (11)$$

The tea was made at around 4pm.

Error Margins and Ethical Considerations:

- Assumptions of constant room temperature – this can vary.
- The tea might not have started at a boiling point.
- External factors may impact cooling rate.
- Incorrect assumptions can result in inaccurate accusations.
- Ethical considerations are essential when modelling real-world events.

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Problem 3 Solution – Mathematics of Military Engagement

Problem 3 – Mathematics of Military Engagement
Topic: Differential equations

1. Setup for Lanchester's Law:

Consider two armies, R (red) and B (blue). Let:

- $m_R(t)$ = Number of soldiers in the red army at time t.
- $m_B(t)$ = Number of soldiers in the blue army at time t.

Assumption: Each army loses soldiers proportional to the strength of the opposing army. This assumption can be described by the following equations:

$$\frac{dm_B}{dt} = -\,a_R m_R \qquad \textbf{(1)}$$

$$\frac{dm_R}{dt} = -\,a_B m_B \qquad \textbf{(2)}$$

The proportionality constants a_R and a_B indicate how effectively each army inflicts losses on the other.
The initial conditions as follows:

$$m_B(0) = m_{B_0} \text{ and } m_R(0) = m_{R_0} \qquad \textbf{(3)}$$

where m_{B_0} and m_{R_0} represent the initial number of soldiers in the blue and red armies, respectively.

2. Derivation of Lanchester's Law:

To solve the system of differential equations, we can use the separation of variables technique:

$$\frac{dm_B}{dm_R} = \frac{a_R m_R}{a_B m_B}$$

$$\Rightarrow a_B m_B \, dm_B = a_R m_R \, dm_R$$

$$\Rightarrow a_B \int m_B \, dm_B = a_R \int m_R \, dm_R$$

Integrating both sides gives,

$$a_B \frac{m_B^2(t)}{2} = a_R \frac{m_R^2(t)}{2} + c \qquad \textbf{(4)}$$

Applying the initial conditions **(3)** to **(4)** above yields :

$$a_B \left(m_B^2(t) - m_B^2(0)\right) = a_R \left(m_R^2(t) - m_R^2(0)\right)$$

This is **Lanchester's Square Law**.

3. Ethical Considerations

- The constants a_B and a_R can be interpreted as the **killing rates** of the two armies.
- They represent how effectively each army eliminates the other's soldiers.
- This piece of mathematics might influence to predict casualties and make strategic decisions.
- Even simple calculus equations can have life and death consequences.

Ethical questions to Consider

- *Would you be comfortable applying these equations in real world military scenarios?*
- *Should mathematicians take responsibility for how their models are used?*
- *How do we balance the pursuit of knowledge with its potential consequences?*

Mathematics is not just a set of abstract concepts and rules. It plays a crucial role in shaping the real world. Therefore, recognising its ethical implications is vital for responsible application of its methods.

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Problem 4 Solution Part a- Mathematical Communication

Problem 4 – Mathematical Communication

Topic: Numerical methods (Newton Raphson Iteration)

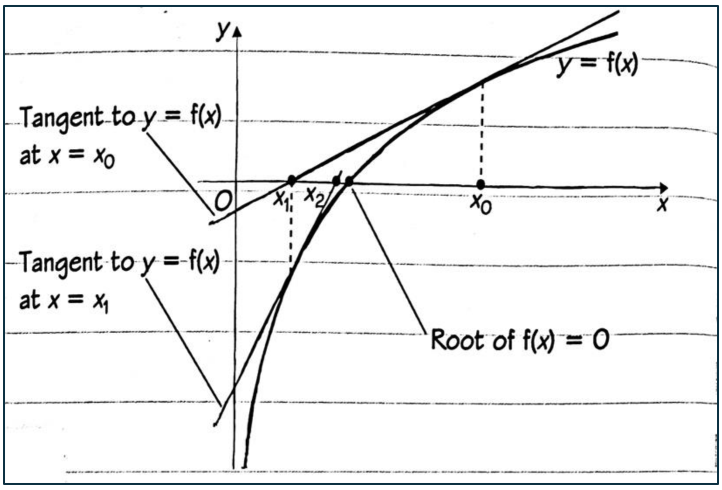
(a) Newton Raphson Iteration:

Newton-Raphson iteration is a method used to find approximations of roots of a function $F(x)$ when an exact solution is difficult to determine. It follows the iterative formula:

$$x_{n+1} = x_n - \frac{F(x_n)}{F'(x_n)}$$

- Starts with an initial guess x_0 between a and b.
- Uses the tangent line at x_n to find a better estimate x_{n+1}.
- Repeats the process, getting closer to the actual solution $x = x^*$.

Graphical Explanation:



- Start with an initial guess x_0 within the interval [a, b].
- Draw the tangent line to the curve of F at x_0.
- Extend this tangent until it intersects the x-axis at a new point x_1, which becomes the next estimate.
- Repeat the process, refining the estimate at each step.

Because $F'(x) > 0$, the function is always increasing, ensuring that this method quickly converges to the correct root $x = x^*$.

Simplified Explanation for Non-Mathematicians.

- Think of a smooth curve that starts below the x-axis and ends above it. In between, it must cross the x-axis and that is its actual root.
- To approximate the root, we start with a reasonable guess and improve it using refined adjustments based on the tangent line (slope of curve) to get a better estimate.
- This process can enable us to get closer to the actual root within a certain degree of accuracy.

Ethical Considerations

- **Simplifying mathematical terminology:** Avoid using advanced terms like ‘derivative’ or ‘convergence’. Instead use familiar language such as ‘rate of change’ for ‘derivative’ and ‘getting closer to the answer’ for ‘convergence’. This ensures a lay audience can understand the explanation.
- **Respect for educational background:** Many people may not have studied beyond high school mathematics. Therefore, avoid discussing complex mathematical properties like ‘smooth functions’ or ‘uniqueness of solutions’ without providing clear, intuitive explanation. Visuals and analogies are key to making the content reliable.

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Problem 4 Solution Part b - Mathematical Communication

Problem 4 – Mathematical Communication
Topic: Numerical methods (Finite Difference Methods)

(b) Finite Difference Methods

Finite difference methods estimate the derivative of a function $F(x)$ at a specific point a using a small step size h. Three common approaches exist:

- **Forward Difference (uses a point ahead of a i.e. $a+h$ to estimate the slope.)**

$$F'(a) \approx \frac{F(a+h) - F(a)}{h}$$

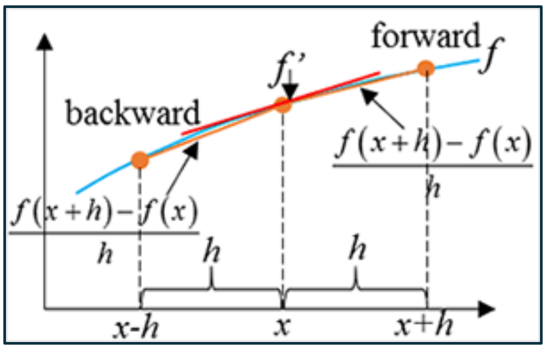
- **Backward Difference (uses a point behind a i.e. $a-h$ to estimate the slope.)**

$$F'(a) \approx \frac{F(a) - F(a-h)}{h}$$

- **Central Difference (uses points on both sides of a i.e. $a+h$ and $a-h$ for a more accurate estimate.)**

$$F'(a) \approx \frac{F(a+h) - F(a-h)}{2h}$$

Graphical Explanation



- Draw the function $\boldsymbol{F(x)}$ and pick a point $\boldsymbol{a}$.
- Draw a tangent line at $\boldsymbol{a}$; this is the slope of the graph $\boldsymbol{F}$ at point $\boldsymbol{a}$, which is $\boldsymbol{F'(a)}$.
- **Forward difference:** Take a second point $\boldsymbol{a+h}$. Connect it to $\boldsymbol{a}$ with a straight line to approximate its slope.
- **Backward difference :** Take a second point $\boldsymbol{a-h}$. Connect it to $\boldsymbol{a}$ with a straight line to approximate its slope.
- **Central difference:** Take two points, one at $\boldsymbol{a-h}$ and the other at $\boldsymbol{a+h}$. Connect them with a straight line. This results in a better approximation to the tangent.
- This is because, the central difference method averages the forward and backward approximations, making it more accurate.

Simplified Explanation for Non-Mathematicians.

- Consider a hill: The slope at any point tells us how steep the hill is.
- Instead of guessing the slope, we calculate it by measuring changes between nearby points.
- The step size h is the distance between the points we measure. A smaller step size gives a more accurate approximate of the slope.
- The forward difference looks ahead to estimate the slope, while the backward difference looks behind.
- The central difference method is the best because it averages the forward and backward estimates, giving a more accurate result.

Ethical Considerations

- **Clarity through visual aids:** Since the concept involves approximating derivatives, it is important to emphasise the visual representation of the slope and avoid mathematical symbols like $\boldsymbol{F'(a)}$ or **step size $\boldsymbol{h}$** without context.
- **Inclusive communication:** Using terms such as 'forward difference', 'backward difference', or 'central difference' might confuse the audience. Replace these with simple phrases such as 'looking ahead 'or 'looking behind' to make the methods more relatable and easier to understand.

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