Queen Mary
University of London

Teaching Ethics and Sustainability in Mathematics?

You Must be Joking!

Dr. Rehan Shah

Lecturer (Assistant Professor)

in Mathematics and Engineering Education,

Queen Mary University of London

Ethics in Mathematics: An Overview

- Why do we need it?
- What is its current state?
- How do we teach it? (*interactive discussions*)
- Some ‘typical’ responses

Need for Ethics in Mathematics (EiM)

- **Both** pure and applied mathematics can lead to ethical consequences, mathematics today is an extremely sharp **double-edged sword**

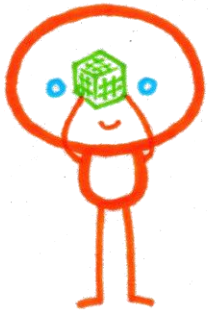
- Many examples of this such as **financial mathematics, data science, AI, statistics, mass surveillance, social networks, industrial mathematical modelling, cryptography, communication metadata, etc.**

- Mathematicians are **uniquely** responsible for the immediate **moral, ethical and legal consequences** of their work

- As educators, we train professional mathematicians, but are we really giving them a **professional training?**

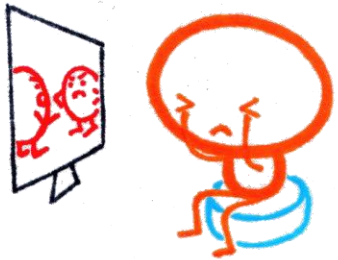
Current state of EiM: Not my problem!

This is M.



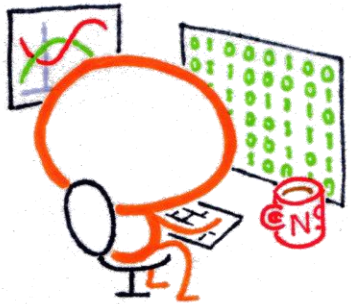
M is very clever and likes solving fun puzzles and maths problems.

M doesn't like thinking about politics or society.



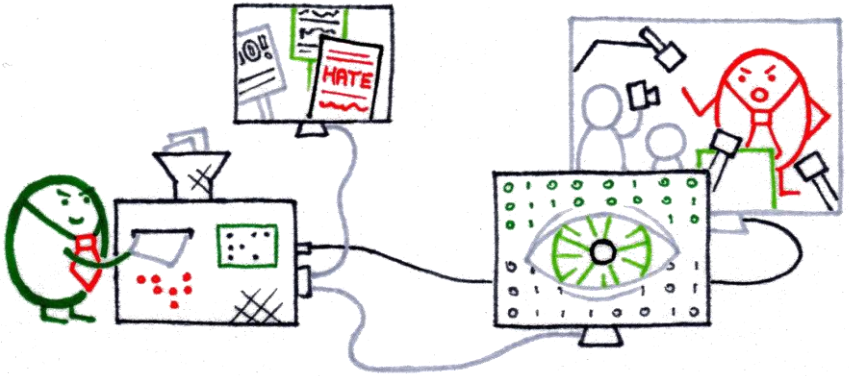
The outside world is scary.

At work, M is given shiny new problems to work out every day.



M is well looked after, and doesn't have to confront the outside world at all.

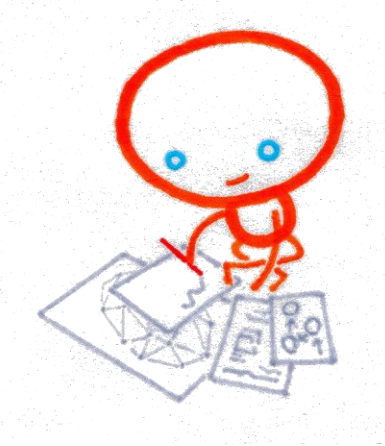
Current state of EiM: Not my problem!



And who would want to anyway?

The news seemed to be getting worse every day, which made M feel worried and scared.

Better far to ignore it all, and get back to that interesting problem…



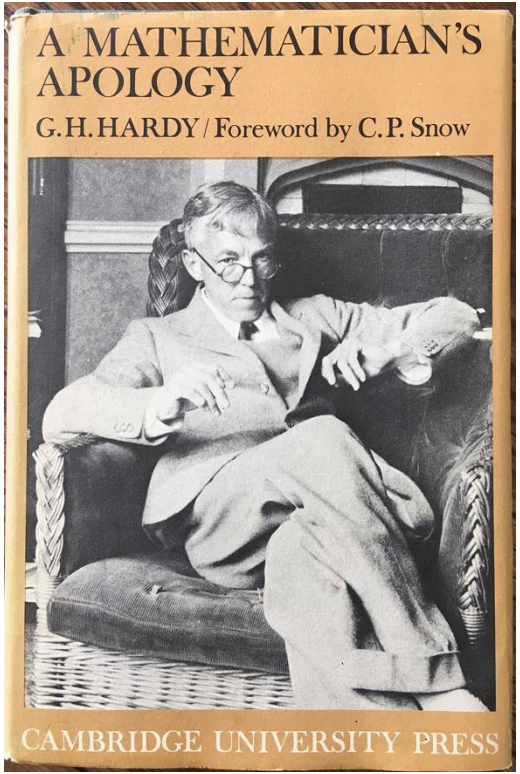
Made by Phoebe Young
www.ethics.maths.cam.ac.uk

Professional Views of EiM

- Mathematical community often still believes that while mathematics might be applied to various situations with social consequences, it is itself *free of ethical considerations*.

"No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world."

- ***G.H. Hardy,*** *A Mathematician's Apology* (1940)



- The *American Mathematical Society's* (AMS) ethical policy statement (https://www.ams.org/about-us/governance/policy-statements/sec-ethics) is largely **concerned only with academic ethical issues** (publishing, reviewing, discrimination)

How can we teach EiM?

- **Interweave ethics into existing mathematics-based courses**

- involves **inserting ethical perspectives into standard mathematics courses**

(e.g. EPC's engineering ethics toolkit https://epc.ac.uk/resources/toolkit/ethics-toolkit/)

- usually done through **careful design of 'tutorial' questions** on weekly problem sheets

(will require training academic staff, as nobody has done this before)

- possibly easier for some (applied?) subjects, less so for others (pure ones?)

(not necessarily – e.g. *Cambridge Maths Tripos* questions from university's '*EiM'* project*)*

https://ethics.maths.cam.ac.uk/assets/teaching/Ethics_questions.pdf

Embedding EiM: Example Problem 1

Exercise 1: Confronting Your Boss with Logic

Topic: Propositional Logic

Ethical Pillars: 5 (Mathematisation of problem)

(a) Consider the statement: 'If you don't do it, then someone else will.'

Express this statement in symbolic notation. Find its contrapositive and its negation, giving each both in symbols and in words.

(b) Your boss has given you a task. The task is well within your technical capability, but you are not sure whether it would be legal or ethical. You feel uneasy, but your boss tells you: 'If you don't do it, then someone else will.'

Do you think that the boss' argument is cogent? Does that depend on who you are or what the project is? How would you answer your boss?

Embedding EiM – Example Problem 1 Commentary

Solution and comments: *The point of this question is to get the students to show, mathematically, that the paradigm "If you don't do it, someone else will" is quite flawed as a argument or incentive to do something. Hopefully this can equip the students with enough realisation to push back on such an argument, should they ever encounter it in their professional life.*

This argument "If you don't do it, someone else will" is flawed because it *assumes* that the activity will happen. But that it not a given.

Compare this to the scenario where there is 1 red sock and 9 blue socks in a bag. 10 people are each invited to come a pick out a sock. Then the statement "If you don't pick the red sock, someone else will" is true, IF all 10 people take up the offer to pick out a sock (because all scenarios are accounted for). But if not all 10 pick out a sock, then "If you don't pick the red sock, someone else will" is no longer true. What is true instead is that "If you don't pick the red sock, someone else *might*". Or they might not.

So the statement "If you don't do it, someone else will" *assumes* that at least one person on earth will carry out the task. But this is not necessarily true. Perhaps you are the only person capable of doing it, in which case it is clearly false. Or perhaps the people capable of doing it (eg: your work colleagues) all refuse, in which case if you don't do it, it will not be done.

Embedding EiM: Example Problem 2

Exercise 2: Exploitation of Knowledge Asymmetry

Topic: Group Theory

Ethical Pillars: 1 (Questioning the product/service), 9 (Mathematical politics)

The *15-puzzle* consists of 15 small square tiles, numbered 1 to 15, which are mounted in a 4×4 frame in such a way that each tile can slide vertically or horizontally into an adjacent square (if it is not already occupied by another tile), but the tiles cannot be lifted out of the tray. In the early 20th century, a cash prize was offered for a solution to manoeuvre the tiles from the first to the second of the configurations shown below.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

15	14	13	12
11	10	9	8
7	6	5	4
3	2	1	

(a) Give such a solution, or show that none exist.

(b) Outline the merits and drawbacks of offering a prize for such a puzzle. Are there any modern-day instances of scenarios where people use their mathematical knowledge to gain (financial, or other) advantage over others?

Embedding EiM: Example Problem 2 Commentary

Solution and comments: *The idea of this question is to prompt the students into asking the question "Should I be using my understanding of mathematics to trick people?" This is a fairly harmless example, but as soon as we consider such puzzles as being sold for profit, then it becomes an issue of exploiting knowledge asymmetry. Of course, someone with sufficient mathematical training would realise that the puzzle is impossible, but not everyone has a mathematics degree; students may not be actively aware of this. A version of this was actually created and cunningly used by the American riddler Sam Loyd (1841-1911) with a prize of $1000 offered to anyone who solved it, and of course no one could*[26]. *Mathematics is used extensively in the modern world to gain advantage over others, such as in stock market trading (in particular, high-frequency trading).*

Embedding EiM: Example Problem 3

Exercise 3: Mathematics of Military Engagement

Topic: Differential Equations

Ethical Pillars: 1 (Questioning the product/service), 9 (Mathematical politics)

Consider two armies R (red) and B (blue). Let $m_R(t)$ and $m_B(t)$ be the number of soldiers of army R and B respectively.

(a) Assume that the losses of each army are proportional to the strength of the other army with proportionality constants a_R and a_B. Set up a system of differential equations that describe the strength of each army at time t.

(b) Derive Lanchester's square law

$$a_B\left(m_B^2(t) - m_B^2(0)\right) = a_R\left(m_R^2(t) - m_R^2(0)\right).$$

(c) How would you describe the proportionality constants a_R and a_B in everyday language? Would you be comfortable working on such problems in your career?

Embedding EiM: Example Problem 3 Commentary

Solution and comments: *This question is designed to show that abstract mathematics can be very "real", and can have very substantial life-and-death consequences, once you put it into everyday language. Here we have mathematics for military engagement; this is very real, and very serious.*

(c) The coefficients a_B and a_R can be interpreted as the killing rate of the armies. This piece of mathematics might influence whether a general sends their army into battle or not, by analysing whether "More of their troops will die than our troops" (a_B and a_R are "death rates" when viewed by the other side). Students may have never considered the fact that even a 5-line piece of calculus can have such a significant impact on warfare (and thus on human life). There is no right or wrong answer to the last part of this question; it only serves to *initiate* these sorts of considerations among students, and should be discussed with open-minded, thoughtful, and non-judgemental discourse.

'Thought-Provoking' Ethics in Mathematics Questions (RS)

Motivation

The mathematical exercises provided below are designed to raise ethical awareness and imbibe transferable skills a
STEM disciplines as part of an ongoing scholarship research initiative undertaken by Dr. Rehan Shah from QMUL wit
of Cambridge. Some of the mathematical content in these exercises was inspired by actual tutorial sheet questions
course taught at the University of Cambridge.

In addition to traditional mathematical content, each of the exercise questions below also contain some ethical con
order to solve the problem fully, you will need to take into account this ethical aspect and consider it as part of your

"Solutions" to as many of these exercises as possible have been provided. These include both a full exposition of th
question, as well as a discussion and incorporation of the societal and ethical issues that are embedded in the quest
points for further discussion with Dr. Shah and your peers.

If you are more interested in this area and would like to engage in it further, please get in touch with Dr. Shah.

Exercise Questions

1. Confronting Your Boss with Logic
2. Ethics of Crime Scene Investigation
3. Mathematics of Military Engagment
4. Ethics of Environmental Disasters
5. Optimisation of Pipeline Construction
6. Mathematical Communication
7. Simpson's Paradox

Task

Please engage and have a look through these materials and then provide your feedback by comple
29 March 2024).

Introductory Handout: Why do we need ethics in mathematics? 172.2 KB

Ethics in Mathematics Exercise Questions 891.5 KB

Ethics in Mathematics Exercise Solutions

Available from **12 February 2024, 9:00 AM**

Feedback Survey

Feedback Survey Questions

Were you aware of "ethics in mathematics" before coming across these problem exercises?

What did you find most enjoyable or interesting about these exercises?

What did you find most challenging or difficult about these exercises?

How did you find the difficulty level of the exercise problems?

Did you find the solutions provided to be sufficient for understanding?

Have these exercises made you want to learn more about ethics in mathematics or the impact of mathematics in the real world?

What suggestions, if any, do you have for improving these mathematical exercises?

Would you like such exercise problems to be introduced in a classroom setting e.g. in lectures or tutorials?

Some ‘typical’ responses

- **“Ethics is just an opinion; it is not mathematically precise”**

Yes, it is subjective, but still important and useful (medical, legal, professional ethics in other disciplines are also not absolute, but does not mean it should not be considered)

- **“This is not my job”**

Yes, it is, then whose job is it? (management/legislators do not have the skills to interpret)

- **“I see no other mathematicians doing this, so why should I?”**

Just because not seen anyone do it, does not mean it should be done (original research?)

- **“What qualifies a non-philosopher to speak about ethics in mathematics?”**

no existing qualifications in ethics, we do not rely philosophers to lay out medical, legal or engineering ethics, doctors speak about ethics, why not mathematicians?

Sustainability in Mathematics: An Overview

- Why do we need it?
- How do we integrate it?
- How do we teach it? (*interactive discussions*)
- Concluding remarks

Need for Sustainability in Mathematics (SiM)

- **Policy requirements:**

QAA Subject Benchmark Statement: Mathematics, Statistics and Operational Research (March 2023)

The UK Standard for Professional Engineering Competence (UK-SPEC) Fourth Edition (August 2020)

Sustainability

1.44 MSOR has a vital role to play in achieving the UN's Sustainable Development Goals, underpinning many technological, scientific and digital developments which have potential to improve health, drive economic growth, transform societies and enhance our environment. For example, mathematical models inform forecasts of climate change, analysis of health data informs public health provision and algorithms help users optimally navigate transport networks. Policies which encourage sustainable development and reduce inequalities can be developed and analysed based on mathematical models and data analysis. MSOR degrees are themselves a driver of social mobility with many graduates from a range of socio-economic backgrounds earning high incomes.

Chartered Engineer (CEng)

E. Personal and professional commitment

Chartered Engineers shall demonstrate a personal commitment to professional standards, recognising obligations to society, the profession and the environment.

The applicant shall demonstrate that they:

1. Understand and comply with relevant codes of conduct
2. Understand the safety implications of their role and manage, apply and improve safe systems of work
3. Understand the principles of sustainable development and apply them in their work
4. Carry out and record the Continuing Professional Development (CPD) necessary to maintain and enhance competence in their own area of practice
5. Understand the ethical issues that may arise in their role and carry out their responsibilities in an ethical manner.

How do we integrate SiM?

Basic principles:

- **Seamless and organic, not artificial and contrived**

- Focus on how students engage with sustainability, do not treat as an add-on component,
- Ensure that concepts emerge naturally from the technical problem by aligning to a realistic scenario

- **Subjective and reflective aspects**

- answers will involve open-ended discussions unlike the usual objective 'right or wrong' answers

- **Link with** UN Sustainable Development Goals

- map concepts to the 17 SDGs part of the 2030 Agenda for Sustainable Development, to give students a more holistic view of sustainability within the context of their discipline

UN Sustainable Development Goals

Embedding SiM: Example Problem 1

Exercise 1: Pipeline Construction

Topic: Optimisation

SDG Mapping: 7 (Affordable and Clean Energy), 9 (Industry, Innovation and Infrastructure), 14 (Life Below Water), 15 (Life on Land)

An oil company wants to build a pipeline connecting an oil platform to a refinery (on land). The coastline is straight. The oil platform is at a distance of 13km from the coast. The refinery is on the coastline, a distance 10km from the point on the coast closest to the platform. Building the pipeline will lead to a cost of £90,000 per km at sea and £60,000 per km on land.

Calculate the optimal length for building the pipeline. What are the factors that need to be considered when providing a response to this question?

Embedding SiM – Example Problem 1 Commentary

Solution comments*: The cost-minimising path is given by Snell's law and is an exercise in trigonometry and calculus. But who said we were optimising over cost? This is an assumption often engrained into mathematicians, engineers, scientists and economists, while they are students, but it need not always be the right way to optimise. How many decisions made by government agencies (often based on advice offered by mathematical consultants) use economics as the sole criterion for optimization?*

Economic actions almost always have externalities, such as possible damage to the environment (the pipe may go through a coral reef or protected habitat) or to existing infrastructure (it may go through a school or a site of archaeological significance). How could we mathematically model the environmental and human impact of laying this pipe? There are numerous factors to consider and students, much like policymakers would, should take a holistic view of these effects and at least be aware of, and question the implications of basing decisions solely on economic factors.

Embedding SiM: Example Problem 2

Exercise 2: Environmental Disasters

Topic: Differential Equations

SDG Mapping: 6 (Clean Water and Sanitation), 12 (Responsible Consumption and Production)

A chemical accident took place near a small village in Peru. The region's local water reservoir has a volume *V.* The inflow and outflow of the reservoir is given by the flow rate *r.* Let *x(t)* be the amount of mercury in the reservoir at time *t.* Assume that the reservoir was clean at the beginning i.e., *x(0) = 0*. Let *C(t)* be the concentration of mercury flowing into the reservoir.

a) Set up and solve a differential equation describing the concentration of the reservoir.
b) How can you use your solution to model repeated pollution (e.g., criminals dumping mercury near the reservoir every weekend)?
c) What are some relevant questions you can ask about the concentration of mercury in the reservoir?
d) Suppose that the polluter is caught and after some cleaning, the incoming water is clean. How can you use your model to analyse when the water in the reservoir will be safe again? How sure are you of your answer and how much does it matter?

Embedding SiM – Example Problem 2 Commentary

Solution comments: *This question is designed to show students that very simple mathematics can be used to model local environmental disasters, which can often be an example of how it may be used unsustainably. It teaches students to find good questions instead of merely answering someone else's questions.*

For part c), possible questions for students to consider can include:

- *Will the pollution of the reservoir ever reach a dangerous level?*
- *What is deemed a "safe" level of mercury in the reservoir?*
- *How closely does the concentration of the reservoir follow the inflow of pollutant chemicals?*
- *Will the reservoir reach an equilibrium concentration of mercury?*

For part d) for the sub question "How much does it matter?", students should identify that we are dealing with poison in drinking water, so it matters immensely! They should understand that this is an estimate, which helps forecast when the water might be safe to drink (the only way to actually know is to thoroughly test it). This question helps students to realise that the mathematics is simply one part of a much bigger solution and should not be relied upon as a definitive answer to a question as serious as the safety of drinking water.

Embedding SiM: Example Problem 3

Total:

	Applications	Successful
Female	300	30
Male	1000	210

Exercise 3: Simpson's Paradox

Topic: Probability

SDG Mapping: 5 (Gender Equality), 10 (Reduced Inequalities)

Prefer applied:

	Applications	Successful
Female	270	18
Male	350	15

Prefer pure:

	Applications	Successful
Female	30	12
Male	650	195

In a particular admissions cycle, a mathematics department observes a higher success rate for male applicants than for female applicants. To investigate whether this is the same across the two sub-departments of Pure Mathematics and Applied Mathematics, the following year the department asks each applicant to give their preference for pure or applied mathematics (they are not allowed to be ambivalent) and records the resulting statistics as shown in the figure below:

a) Compare the success rates for male and female applicants that prefer applied mathematics, prefer pure mathematics and their success rates overall.
b) What do you notice? Why is this possible? This is known as Simpson's Paradox.
c) If possible, find the admission statistics by gender and mathematics preference (pure/applied) from your university's mathematics department and see if the same phenomenon occurs.

Embedding SiM – Example Problem 3 Commentary

Solution comments: *The purpose of this question is to demonstrate Simpson's paradox in which a trend appears in several different groups of data, but disappears or reverses when these groups are combined. It also attempts to highlight the immense gender disparity in many mathematics departments around the world.*

(a) We calculate the success rates:

	Prefer applied	Prefer pure	Total
Female	$\frac{18}{270} = \frac{14}{210}$	$\frac{12}{30} = \frac{4}{10}$	$\frac{30}{300} = \frac{10}{100}$
Male	$\frac{15}{350} = \frac{9}{210}$	$\frac{195}{650} = \frac{3}{10}$	$\frac{210}{1000} = \frac{21}{100}$

(b) We note that females with a given preference have a higher success rate than males with the same preference, but lower overall. This is Simpson's Paradox.

(c) Students may have access to this data. The main point of the question is not so much to redo the calculation (it is not a given that Simpson's paradox will always arise here), but rather to illustrate the immense gender disparity in many mathematics departments around the world, which is likely (but not certain) to include the department the students are currently in. Observations like this resonate more when they are closer to home.

Concluding remarks

- **Faculty support is therefore critical** for integrating EiM and SiM into courses, but this can be hard (responses such as "*we're a mathematics department or this is a technical mechanics module, why are we teaching ethics/sustainability?"*)

- Ethics/sustainability *is* a matter of opinion, but that **does not mean it cannot be addressed**

- When other disciplines face ethical/sustainable issues and train professionals to deal with them, **how can we exclude ourselves from them?**

- As educators, we have a **duty to teach our students** how to use the mathematics they learn responsibly

References

Ethics in mathematics resources: Cambridge EiM project website

- Ethics in Mathematics Teaching Toolkit, M. Chiodo, D. Muller, R. Shah, arXiv, 2023.
- The role of ethics in a mathematical education, M. Chiodo, R. Vyas, Ethics in Mathematics Discussion Papers, 2019.
- Teaching Ethics in Mathematics, M. Chiodo, P. Bursill-Hall, LMS Newsletter **485**, 22-26, November 2019. Republished in the EMS Newsletter **114**, 38-41, December 2019

Embedding sustainability in STEM subjects are available from the EPC's Sustainability Toolkit

- EPC Sustainability Toolkit Guidance Article, R. Shah (soon to appear in 2024).
- Incorporating the sustainable development goals in engineering education, R.A. Ramirez-Mendoza, R. Morales-Menendez, E.M. Melchor-Martinez, *Int J Interact Des Manuf* 14, 739–745, 2020.


Thank you for your time!
Queen Mary
University of London